16/Eng06/1081

If $y = e^{e^x} + x$

$V = x^2 + 2$

$\frac{dy}{dx} = 2x + 1$

$y = e^V$

$\frac{dy}{dx} = e^V$

$\frac{dy}{dx} = \frac{dy}{dV} \times \frac{dV}{dx}$

$= e^V \times 2x + 1$

$2x + 1e^V \; U = x^2 + x$

$\frac{dy}{dx} = 2x + 1e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2ex^2 + x + (2x+1)(2x+1)e^{x^2} + x$

$\frac{d^2y}{dx^2} = 2e^{x^2} + x + 4x^2 + 4x + 1 + e^{2x} + x$

$y'' = \frac{d^2y}{dx^2} = y' + \frac{dy}{dx} \quad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x2+x} + 4x^2 + 4x + 1e^{x2+x}$

$y' = (2x+1) = (2x+1)(2x+1)e^{x2+x}$

$= 4x^2 + 4x + 1e^{x2+x}$

$2y = 2e^{x2+x}$

$y'(2x+1) + 2y = 2e^{x_2+x} + 4x^2 + 4x + 1e^{x2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1ex^2 + x$

$y'' = y'(2x+1) + 2y$

$w_1 \quad w_2 \quad w_3$

w_1

$V = y'' \qquad V = 1$

$V^n = y^{n+2} \qquad V = 0$

$= y^{n+2} - 1 + 0$

w_2

$U = y' \qquad V = 2x + 1$

$U^n = y^{n+1} \qquad U = 0$

$(2[y^n - 1] + 0$

$w_1 = w_2 + w_2$

$y^{n+1} = y^{n+1}[2x+1) + 2n(yn) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

(28) Using the Leibnitz theorem

$y = x^3 e^{4x}$ det $y^{(5)}$

Soln

$y^{(5)} = U^{(5)}V + 5U^4V' + 10U^3V'' + 10U^2V^3 + 5U'V^4 +$

$= 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x} - 3x^2) + 10(4^3 e^{4x}(6x) + 5(4^2 e^{4x} \cdot 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} 3x^2 + 640 e^{4x} \cdot 6x + 30 e^{4x} \cdot 6$

$= 1024 e^{4x} x^3 + 3840 e^4 x^2 + 3840 e^{4x} x + 480 e^{4x}$

$x^2 \frac{dy^2}{dx} + x\frac{dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

$w_1 \quad w_2 \quad w_2$

For w_1

$U = y'' \qquad V = x^2$

$U^n = y^{n+2} \qquad V' = 2x$

$V^{n-1} = y^{n+1} \qquad V'' = 2$

$= y(n+2)(x^2) + nC(y^{n+1})2x + nC(n-1)\,y^n \cdot x = 0$

$= x^2 y^{(n+2)} + 2nx(y^{n+1}) + nC(n-1)y^{n2}$

For w_2

$w = y_1 \qquad U = x$

$U^n = y^{n+1} \qquad V = 1$

$V^{n-1} = y^n \qquad U'' = 0$

$= y^{n+1} ex + ny^n + 0$

For w_3

$V = y \qquad V = 1$

$V^n = y^n \qquad V' = 0$

$= y^n$

$w_1 + w_2 + w_3 = 0$

$x^2 y^{n+2} + 2nx y^{n+1} + n(n^2 - n)y^n + xy^{n+1} + ny^n + y^n$

$x y^{n+2} + 2n^{x} y^{n-1} + xy^{n+1} + n^2 y^n - ny^n + ny^n + y^n$

$x^2 y^{n+2} + 2n+1\,(2x y^{n+1}) + (n^2+1)y^n$